Second-Moment Statistical Approach for Input Uncertainty Propagation and Robust Design Using CFD

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ASCoT Project (1998-2002)

Aerospace Systems (Concept to Test)

Project Vision

Physics-based modeling and simulation with sufficient speed and accuracy for validation and certification of advanced aerospace vehicle design in less than 1 year

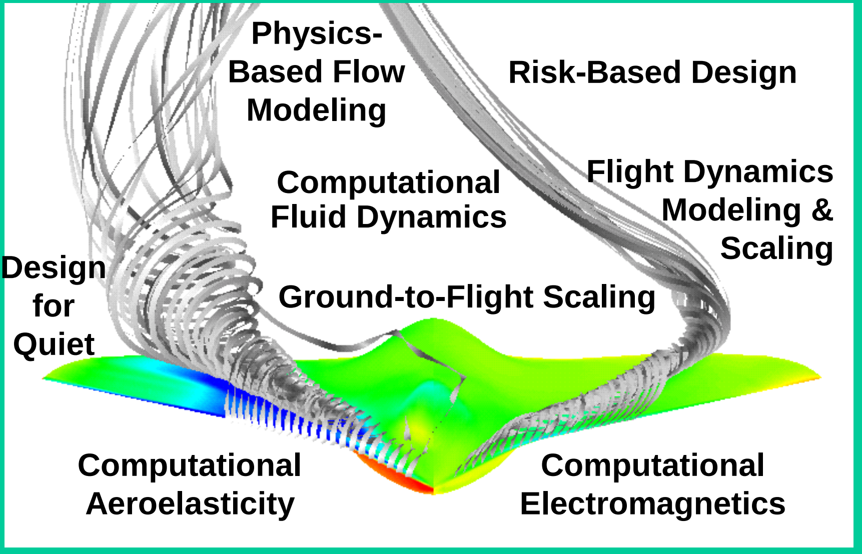
Project Goal

- **Provide next-generation analysis & design tools to increase confidence and reduce development time in aerospace vehicle designs**

Objective

- Develop **fast, accurate, and reliable** analysis and design tools via fundamental technological advances in:
  - Physics-Based Flow Modeling
  - Fast, Adaptive, Aerospace Tools (CFD)
  - Ground-to-Flight Scaling
  - Time-Dependent Methods
  - Design for Quiet
  - Risk-Based Design w/ CFD & CSM

Technology Areas



Benefit

- Increased Design Confidence
- Reduced Development Time

Outline

- **Assumptions**
- **Second-moment statistical approach**
- **Quasi 1-D nozzle, Euler CFD**
  - **Input uncertainty propagation**
  - **Robust optimization**
- **Conclusions**

Assumptions

- **Only input parameter uncertainties contribute to output uncertainties; i.e., other uncertainty sources not considered**
- **Input parameters are**
  - **Statistically independent**
  - **Random**
  - **Normally distributed with**
    - **Mean values,** $\overline{b_i}$
    - **Standard deviations,** σ_{b_i}

1st and 2nd- Order Taylor Series Approximations for Output F(b)

First-Order:
(FO)

$$\mathbf{F(b)} = \mathbf{F(\bar{b})} + \sum_{i=1}^{n} \frac{\partial \mathbf{F}}{\partial b_i}(b_i - \bar{b}_i)$$

Second-Order:
(SO)

$$\mathbf{F(b)} = \mathbf{F(\bar{b})} + \sum_{i=1}^{n} \frac{\partial \mathbf{F}}{\partial b_i}(b_i - \bar{b}_i) + \\ + \frac{1}{2!}\sum_{j=1}^{n} \sum_{i=1}^{n} \frac{\partial^2 \mathbf{F}}{\partial b_i \partial b_j}(b_i - \bar{b}_i)(b_j - \bar{b}_j)$$

where all derivatives are evaluated at the mean values, $\bar{\mathbf{b}}$.

- Note that efficient first- and second-derivatives are needed from CFD codes

Approximate Mean and Variance

FO FM: $$\overline{\mathbf{F}} = \mathbf{F}(\overline{\mathbf{b}})$$

FO SM: $$\sigma_{\mathbf{F}}{}^2 = \sum_{i=1}^{n} \left(\frac{\partial \mathbf{F}}{\partial b_i} \sigma_{b_i} \right)^2$$

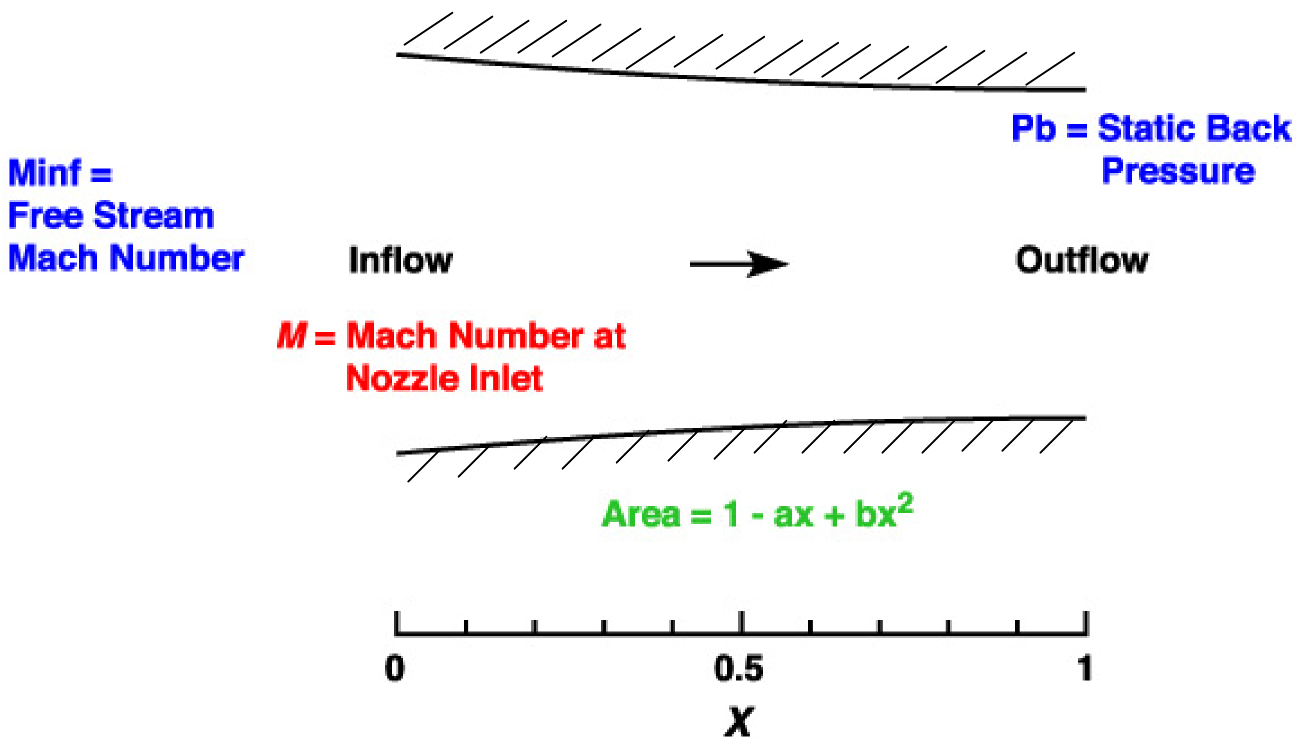
SO FM: $$\overline{\mathbf{F}} = \mathbf{F}(\overline{\mathbf{b}}) + \frac{1}{2!} \sum_{i=1}^{n} \frac{\partial^2 \mathbf{F}}{\partial b_i^2} \sigma_{b_i}^2$$

SO SM: $$\sigma_{\mathbf{F}}{}^2 = \sum_{i=1}^{n} \left(\frac{\partial \mathbf{F}}{\partial b_i} \sigma_{b_i} \right)^2 + \frac{1}{2!} \sum_{j=1}^{n} \sum_{i=1}^{n} \left(\frac{\partial^2 \mathbf{F}}{\partial b_i \partial b_j} \sigma_{b_i} \sigma_{b_j} \right)^2$$

Quasi 1-D Euler Problem

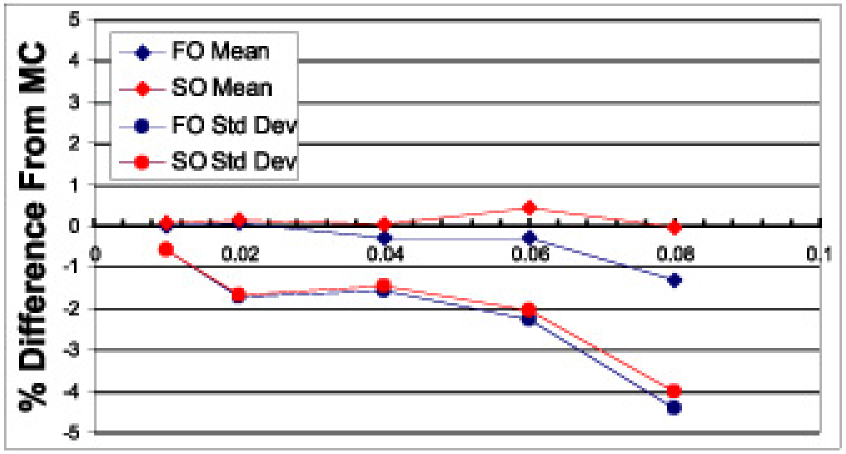
Subsonic Nozzle Flow

Input Random Variables:		CFD Output Function:
Geometric	b={a,b}	F={M}
Flow	b={Minf, Pb}	

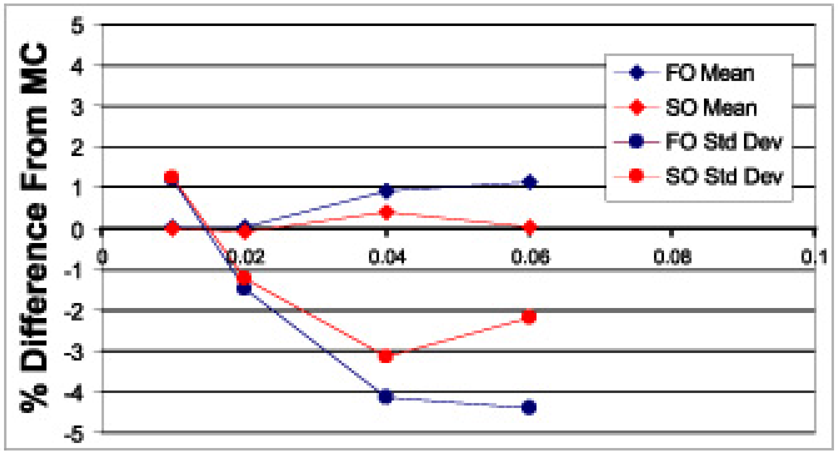


Comparison of Statistical Approximations vs. Monte Carlo Simulation

Geometric Example

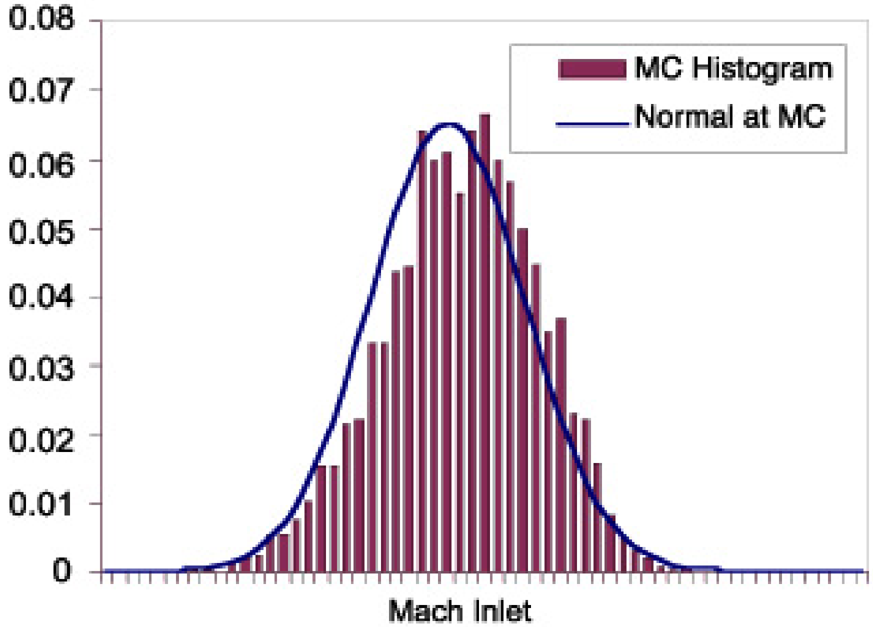


Flow Parameter Example

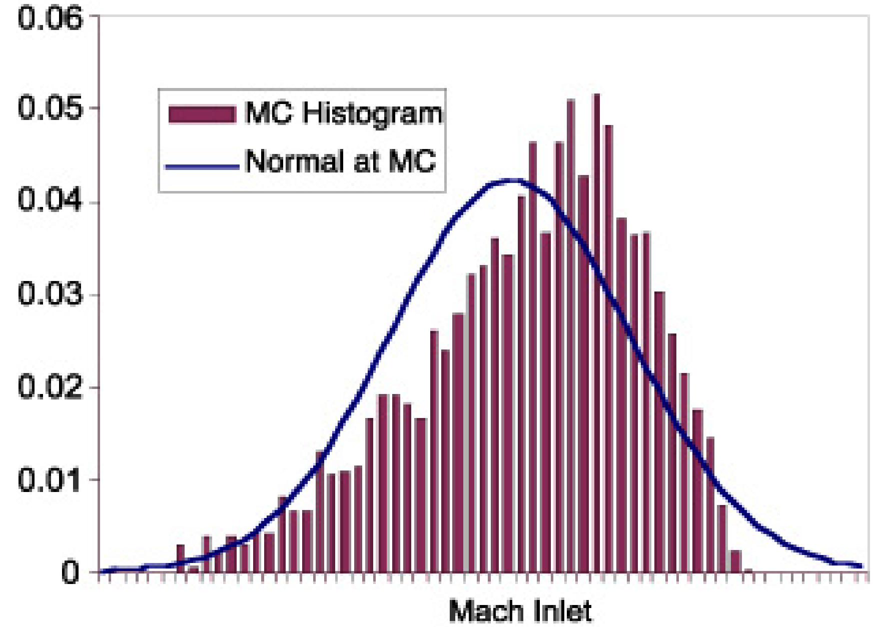


- For larger values of input parameters, second-order generally gives better predictions
- Approximations predict first moment more accurately than second moment

Probability Density Functions from Monte Carlo



$\sigma_{Minf} = \sigma_{Pb} = 0.02$



$\sigma_{Minf} = \sigma_{Pb} = 0.06$

- The Monte Carlo simulation histograms are compared with a normal distribution using the mean & standard deviation from the Monte Carlo simulations (graphically indistinguishable from FOSM & SOSM)
- The FOSM & SOSM results appear adequate for robust design but not for reliability-based design

Robust Optimization

Objective function uncertain due to uncertain input variables

min Obj, $\quad \mathrm{Obj} = \mathrm{Obj}(\overline{\mathbf{F}}, \sigma_{F}, \overline{\mathbf{Q}}, \overline{\mathbf{b}})$

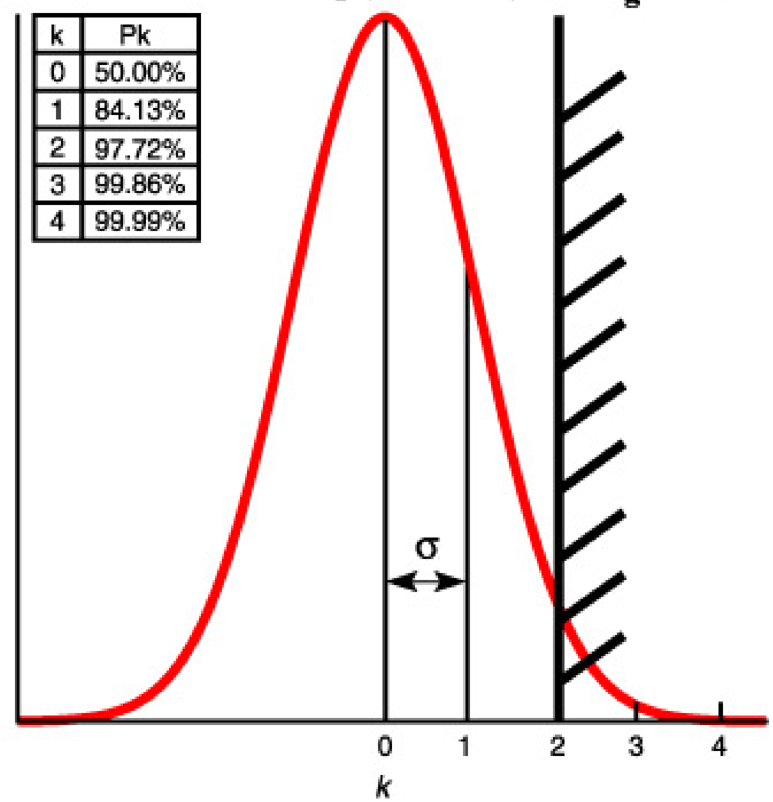
subject to

$$\mathbf{R}(\overline{\mathbf{Q}}, \overline{\mathbf{b}}) = 0$$

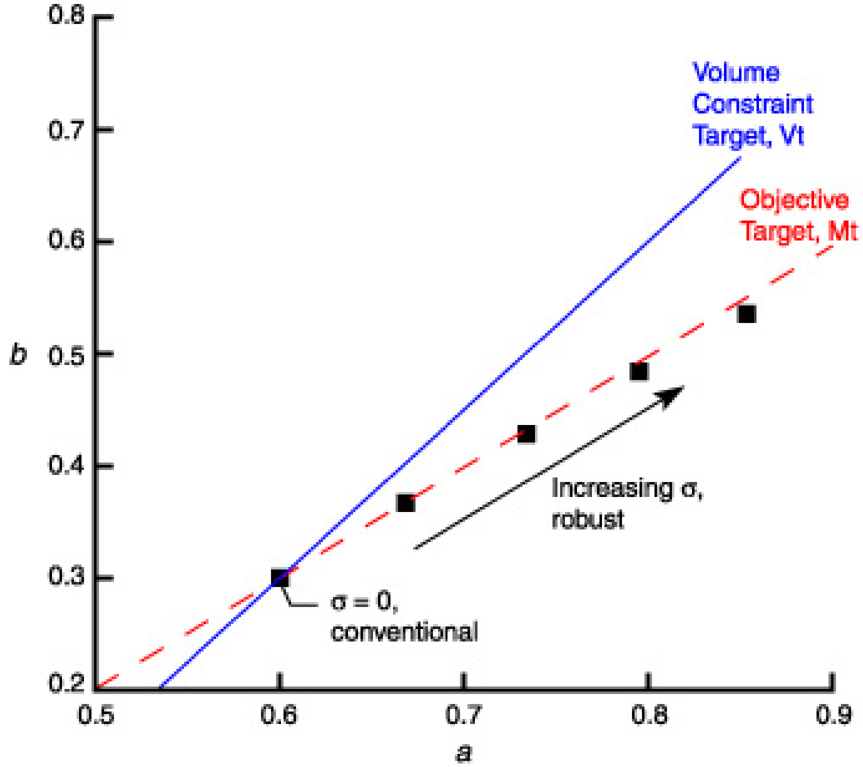
$$\mathbf{g}(\overline{\mathbf{F}}, \overline{\mathbf{Q}}, \overline{\mathbf{b}}) + k\sigma_{\mathbf{g}} \leq 0$$

- $k\sigma_{\mathbf{g}}$ represents the desired safety factor for probabilistic constraint satisfaction

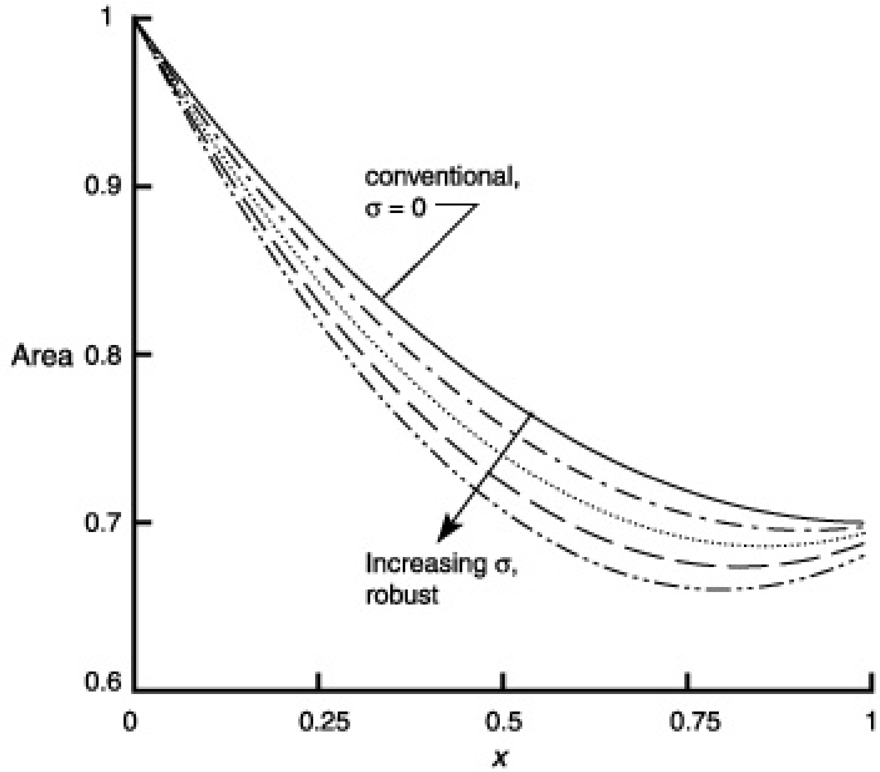
$$\mathbf{g}(\overline{\mathbf{F}}, \overline{\mathbf{Q}}, \overline{\mathbf{b}}) + 2\sigma_{\mathbf{g}} \leq 0$$



Robust Shape Optimization Results, Increasing σ, $P_k = P_1 = 84.13\%$



Design Space



Nozzle Area Distributions

Conclusions

(from present initial results)

- **Demonstrated implementations of second moment approximate statistical method for input uncertainty propagation and gradient-based robust design in CFD**
- **Second-moment method appears applicable for robust design at reasonable uncertainty levels in random input parameters**
- **Propagated uncertainty appears as a probabilistic “safety factor” on constraints; when active, these constraints influence the optimization more than the propagated uncertainty appearing in the objective**

References

1. Sherman, L., Taylor III, A., Green, L., Newman, P., Hou, G., and Korivi, M., "First- and Second-Order Aerodynamic Sensitivity Derivatives via Automatic Differentiation with Incremental Iterative Methods," *Journal of Computational Physics*, Vol. 129, No. 2, 1996, pp. 307-336.
2. Putko. M. M., Newman, P. A., Taylor III, A. C., and Green, L. L., "Approach for Uncertainty Propagation and Robust Design in CFD Using Sensitivity Derivatives," 15th AIAA CFD Conference, Anaheim, CA, June 2001, AIAA Paper 2001-2528.
3. Taylor III, A. C., Green, L. L., Newman, P. A., and Putko, M. M., "Some Advanced Concepts in Discrete Aerodynamic Sensitivity Analysis," 15th AIAA CFD Conference, Anaheim, CA, June 2001, AIAA Paper 2001-2529.
4. Putko. M. M., Newman, P. A., Taylor III, A. C., and Green, L. L., "Approach for Input Uncertainty Propagation and Robust Design in CFD Using Sensitivity Derivatives," to appear in *Journal of Fluids Engineering*, March 2002, a special issue on quantifying uncertainty in CFD.